

Advanced Manufacturing & Prototyping Integrated to Unlock Potential

STATISTICS AND PROBABILITY

Data-Driven Decision Making

SWEET MACHINES

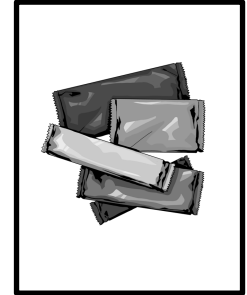
Automated Packaging Challenge

SECTION 1 – THE CANDY AUTOMATED PACKAGING CHALLENGE

1.1 INTRODUCTION

Most items that we buy in stores come packaged or wrapped in something.

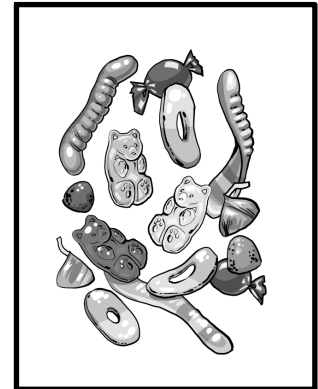
The packaging materials are usually cardboard, paper, or plastic.



Discuss these questions as a class:

1. Why are items at the store individually packaged?
2. Why is most packaging made of plastic, cardboard, or paper?
3. What are some things that you buy that are not pre-packaged? Why do you think they are not pre-packaged?

If you go to some candy stores, you can pick out a selection of candy from bins and the sales clerk might put them into a bag by hand. This allows you to choose exactly what you want, but it is a very slow process.



Today, most things we buy are mass produced on an assembly line and packaged by machines and robots. The advantage of using an assembly line and high tech, computerized robots is that they can make more products and do it more quickly. However, there can be disadvantages as well. One problem is that robots and assembly lines are expensive to buy and install. Company owners need to be sure that they will be able to sell enough of their products to pay for the equipment.

Another effect of automating work with machines, robots, and computers is that fewer people are needed to do the work, which means that there are less jobs available. Workers in highly automated

6DMM Automated Packaging Challenge

plants need to learn how to run the machines and program the computers that tell the machines what to do. These jobs require more skill, education, and training than simple packaging jobs.



Watch *How Candy Canes are Made* video #1.



Discuss these questions as a class:

1. In the video, what are some examples of automation?
2. Which steps are still done by hand?
3. What parts of the packaging process are made easier by technology?

1.2 THE CHALLENGE

Your team has created a new type of small candy that comes in four very different flavors. You want to package the candy in small bags that have equal numbers of each flavor. In order to do this, you will need to purchase a candy-packaging robot to use at your manufacturing plant, but there are many different robots to choose from. You want to make sure you make the right choice. Your customers will probably be happy if there are the same number of each flavor in the bag. Your customers will not continue to buy your candy if some bags have very few of their favorite flavor. Plus, the robots vary greatly in price and some require more workers than others. In order to make an intelligent decision, you will need to collect and analyze some data. So, put on your candy maker's thinking cap, and let's get started.

1.3 CANDY SORT EXPLORATION

Small candies are often packaged in bags that contain a mix of colors and flavors. The best mix is when each bag contains equal numbers of flavors, so people will know what they are getting. Have you ever experienced opening a bag of candy that is not evenly mixed? Sometimes, you may get eight pieces of the flavor or color that you do not like, and only two of your favorite. As you start your candy business, you need to understand candy mixing. Plus, you need a way to judge how well your packaging robot is working. That includes how fast it works and how you can evaluate the **consistency**, or accuracy, of the packaging process.

The goal of this activity is to create four bags of candy with each bag containing approximately the same number of candy pieces and each bag containing all four colors with approximately the same number of each color. You must create each bag as quickly as possible. Your team will receive points for both speed and accuracy. The winning team will be determined based on both measures.

AVAILABLE MATERIALS

- 4 small zip-locked bags of candy with each bag a different color
- Cardboard tray
- Stopwatch
- *Candy Sort Data Collection Sheet*

Part 1 of 2 – Creating the Mixed Candy Bags

Procedure:

1. With your team, brainstorm the steps of the fastest procedure you can think of that will create four bags that are all the same.
2. Write the steps of your procedure on the *Candy Sort Data Collection Sheet*. You are not required to have five steps. Be sure to think through your steps and use as many lines as you need.
3. Assign one team member as the time keeper.
4. When your teacher gives the signal, start the stopwatch and follow your procedure.
5. When all four bags are filled and re-sealed, stop the stopwatch. Keep the candy in the bags.
6. Record the time on your *Candy Sort Data Collection Sheet*.
7. Raise your hand to let your teacher know that you are finished.
8. As each group shares the number of seconds it required to package their bags, record the class data in Data Table #1 on your *Candy Sort Data Collection Sheet*.

6DMM Automated Packaging Challenge

Part 2 of 2 – Collecting Data about Accuracy and Consistency

Procedure (continued):

9. Lay each bag flat and keep the candy in the bags.
10. Count the total number of each color of candy in bag #1.
11. Record the number of each candy color in bag #1 in Data Table #2 on your *Candy Sort Data Collection Sheet*.
12. Repeat steps 9-11 for bag #2, bag #3, and bag #4.
13. Add and record the total number of each color candy in each bag in Data Table #2 on your *Candy Sort Data Collection Sheet*.

1.4 ADD TO YOUR UNDERSTANDING: FIVE-NUMBER SUMMARY

The different teams in your classroom probably had very different results from the candy sorting exploration. Some teams were faster at following their procedure than others. Also, some teams were more consistent with following their procedures than others. Statisticians would say that procedures that produce bags with equal numbers of each color candy have a high degree of **consistency**. Your team will earn points for the time it took to follow your procedure and for the consistency of following your procedure. We need numerical ways to describe how fast and consistent your procedure was.

There are five values mathematicians use to analyze data. Together, these numbers are called the **five-number summary**. Three numbers included in this five-number summary are ones you have seen before.

- **Minimum:** The lowest value in the data set
- **Median:** The number that represents the middle value or center of a data set
- **Maximum:** The highest value in the data set

Two other numbers, which we will use to give us a better picture of how consistent the data is, are the:

- **Lower Quartile:** The median, or center, of the lower half of the data. We will label this as Q1.
- **Upper Quartile:** The median, or center, of the upper half of the data. We will label this as Q3.

The other useful measures are two types of **ranges**, which show you the spread of the set of data. In other words, how different the largest and smallest numbers are from each other and how wide the middle part is.

- **Range:** The difference between the minimum and maximum numbers. To calculate the range, subtract the minimum from the maximum.
- **Interquartile range:** The difference between the lower quartile (Q1) and the upper quartile (Q3). To calculate the interquartile range, subtract Q1 from Q3.

Your team will use all of these statistical values as you analyze your data.

1.5 ANALYZING THE TIME DATA

How fast was your procedure compared to the other teams? Follow the procedure to determine how fast your team was compared to the other teams in your class.

Part 1 of 3 – Find the median time required for all the teams to complete their procedure. The median is sometimes referred to as Q2.

Procedure:

1. In the Data Ordering Box at the bottom of your *Candy Sort Data Collection Sheet*, write each of the numbers your class reported, in order from smallest to largest. Your teacher will demonstrate this to the class.
2. If the number of data points is odd, find the number in the exact center. If the number of points is even, take the average between the two middle numbers.
3. Record the median time on your *Candy Sort Data Collection Sheet*.

6DMM Automated Packaging Challenge

Part 2 of 3 – Find the Lower Quartile (Q1). This is the median of the lower half of the data points.

Procedure:

4. If the number of data points between the smallest number (minimum) and the median point is odd, find the number in the exact center between the two data points. If the number of points is even, take the average between the two middle numbers.
5. Record the lower quartile (Q1) on your *Candy Sort Data Collection Sheet*.

Part 3 of 3 – Find the Upper Quartile (Q3). This is the median of the upper half of the data points.

Procedure:

6. If the number of data points between the median point and the largest number (maximum) is odd, find the one in the exact center between them. If the number of points is even, take the average between the two middle numbers.
7. Record the upper quartile (Q3) on your *Candy Sort Data Collection Sheet*.

1.6 HOW FAST WERE YOU?

Statistics give us a way to describe where a number lies compared to the rest of the group.

Procedure:

1. Use the rules below to record your points on the ribbon on page 1 of your *Candy Sort Data Collection Sheet*.
 - If your time was Q1 or below, then your procedure was slower than most of the class. This will earn you 1 point.
 - If your time was between Q1 and Q3, in the interquartile range, your procedure was around average speed. This will earn you 2 points.
 - If your time was Q3 or higher, then your procedure was faster than most of the class. This will earn you 3 points.

SECTION 2 – ANALYZING YOUR DATA

2.1 ANALYZING THE DATA FOR CONSISTENCY

As the candy maker, you want all packages of candy that are sold by your company to have the same number of candies in the bag. You also want each bag to have approximately the same number of each color candy. How well did the procedure you used do in terms of accuracy and consistency?

Were there the right number of pieces of candy in each bag? Did you have 48 pieces of candy in each bag? If not, what was the range in the numbers?

Procedure:

1. Looking at your data in Data Table #2, determine the maximum number and the minimum number of pieces of candy in each bag.
2. Subtract the minimum number from the maximum number to determine your range.
3. Use the rules below to record your points on the Bag Size ribbon on page 2 of your *Candy Sort Data Collection Sheet*.
 - If your range was 0-2, your bags were nearly exact, and you earn 3 points.
 - If your range was 3-8, your bags were close to being right, and you earn 2 points.
 - If your range was over 8, you probably had some bags that really were not the right size. You earn 1 point.

If your four bags of mixed-color candies were perfectly packaged, you would have 12 of each color candy in each bag. Did this happen? Probably not. How can mathematicians and candy makers describe this variability? Statistics enable mathematicians to communicate how consistent a process is. This will also be the basis for giving points in the candy packaging competition.

6DMM Automated Packaging Challenge

2.2 ANALYZING THE COLOR DATA

For the purposes of this candy competition, you will only analyze the set of numbers that you recorded in Data Table #2 on page 2 of your *Candy Sort Data Collection Sheet* to see how consistent they are.

Procedure:

1. Create a dot plot to visualize your data more effectively and see the spread on page 2 of your *Candy Sort Data Collection Sheet*. Using the individual candy color data from Data Table #2, make a dot for each data point on the graph. Your teacher will demonstrate this technique.
2. Record the data from smallest to largest in the Data Ordering Box on page 2 of your *Candy Sort Data Collection Sheet*.
3. Determine the median (Q2), and record this number on your *Candy Sort Data Collection Sheet*.
4. Draw a line on your dot plot at the median and label it Med on your *Candy Sort Data Collection Sheet*.
5. Determine the lower quartile (Q1). Record the number and draw a line on your Dot Plot labeled Q_1 on your *Candy Sort Data Collection Sheet*.
6. Determine the upper quartile (Q3). Record the number and draw a line on your Dot Plot labeled Q_3 on your *Candy Sort Data Collection Sheet*.
7. Determine the minimum and maximum. Record the numbers and draw lines labeled *Min* and *Max* on your *Candy Sort Data Collection Sheet*.

To determine how consistent your data is, you could use the range as you did to analyze the number of pieces of candy in each bag. However, using the range will not tell you if you have just one “outlier” number (i.e. one that is very different from the rest), or whether your data is scattered all over the place. Instead, we need to look at the interquartile range, which is the difference between Q1 and Q3. It tells us how closely the data is grouped around the median point.

Automated Packaging Challenge 6DMM

Procedure:

8. Determine the range and interquartile range. Review section 1.4 if you need help with how to do this. Record the numbers on your *Candy Sort Data Collection Sheet*.
9. Use the rules below to record your points on the Color Consistency ribbon on page 2 of your *Candy Sort Data Collection Sheet*.
 - If your interquartile range was 0-3, your numbers were all clustered right around 12 and were mostly consistent. You earn 3 points.
 - If your interquartile range was 4-8, your numbers were spread out further from the median, and people buying your candy would need to expect some variation in the bags. You earn 2 points.
 - If your interquartile range was greater than 8, then customers would really notice the variation and would not be happy. You earn 1 point.

Your total point count is a combination of your speed, accuracy and consistency.

Time Points + Bag Size Accuracy Points + Color Consistency Points = Total Points

Procedure:

10. Add up your time points from page 1 of your *Candy Sort Data Collection Sheet*, your bag size accuracy points from page 2 of your *Candy Sort Data Collection Sheet*, and your color consistency points from page 2 of your *Candy Sort Data Collection Sheet*.
11. Record the total points on the Total Points ribbon at the bottom of page 2 of your *Candy Sort Data Collection Sheet*.
12. Report your total to your teacher.



Discuss these questions as a class:

1. Which team's procedure resulted in the best overall score? Which was the fastest? Which was the most accurate and consistent?
2. What type of changes would you make to your procedure if you tried the experiment over again?
3. Watch the video that shows the M&M factory. How do they mix their colors?

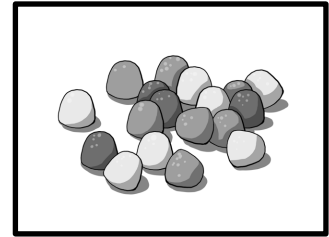


Watch *M&M Manufacturing Plant* video #2.

SECTION 3 – ANOTHER WAY TO ANALYZE DATA

3.1 GRAPHING YOUR DATA: BOX AND WHISKER PLOTS

The five-number summary provides a useful way to describe the different aspects of a set of numbers. Graphs are meant to give a visual summary of data so you can quickly understand it. Mathematicians created a **box and whisker plot** that combines all the five numbers into one graph. It looks a lot like your dot plot that you labeled with lines for the Median, Q1, Q3, minimum and maximum. Your teacher will first demonstrate how to create a box and whisker plot.



Procedure:

1. Transfer your five number summary data from the *Candy Sort Data Collection Sheet* to the *Box and Whisker (Box Plot) Sheet*.
2. On the empty number line on the *Box and Whisker (Box Plot) Sheet*, draw a box and whisker plot with your data, as instructed by your teacher.



Discuss these questions as a class:

1. How can you use a box and whisker plot to quickly judge how consistent a process is?
2. What would it mean about the data if the box is very wide and the whiskers are short?
3. What would it mean about the data if the box is narrow and the whiskers are long?
4. What would it mean about the data if the box is narrow and the whiskers are short?

SECTION 4 – CHOOSING AN AUTOMATED PACKAGING MACHINE**4.1 COMPARING MACHINES**

You now have experience in using data and statistics to judge the accuracy and consistency of a process. As a candy inventor, you will need to make a decision about which candy making robotic machine and packaging equipment to buy. This decision will hopefully allow you to sell a lot of candy.

You need make your decision based on a variety of different data, including:

- **Speed of the process:** How many bags of candy can the robotic machine package per minute? The speed of the process is reported as bags per minute.
- **Accuracy of the packaging:** How accurately does the robotic machine put the right number of pieces of candy in the bag? The accuracy of the packaging is reported as the range between the minimum and maximum bag sizes.
- **Consistency of the flavor mix:** How much variation is there in the mix of candies in each bag? The consistency of the flavor mix is reported as the interquartile range in the flavor mix, and is shown as a box and whisker plot.
- **Expense:** How much does the robotic packaging machine cost? This is reported in dollars.

Four different companies have supplied you with performance data from their robotic packaging machines. This data is located on your *Packaging Company Machines Specifications* student sheet. Each bag was supposed to have 48 candies, with an event distribution of the four colors. The five number summaries provided tell the flavor distribution for candy samples taken from each machine. You also have the minimum and maximum number of candies found in sample bags, the number of bags produced per minute, and the cost of the machine.

Procedure:

1. Draw a box plot that represents the consistency of the packaging of different color candy for each machine.
2. Calculate and record the interquartile range for the color consistency data.
3. Calculate and record the range in number of candies per bag.

Now you have enough data to choose which machine you want to buy for your candy factory.

6DMM Automated Packaging Challenge

4.2 THE DECISION-MAKING TOOL

In order to make a purchasing decision based on data, it is helpful to have all the information in one place. That way you can easily compare the data for different options side-by-side. For this challenge, you will use a **decision-making matrix**. A decision matrix usually includes three categories or types of data. Two categories are things you know, and one category is what you tested for. In this case, we know: 1) the robotic packaging machine company the data was collected from, and 2) the type of data that you collected or were provided with. Your *Decision Matrix* student sheet has those items pre-labeled on the matrix. The third type of data in our challenge, which you will fill into the boxes of the matrix, are the actual numbers for each robot company.

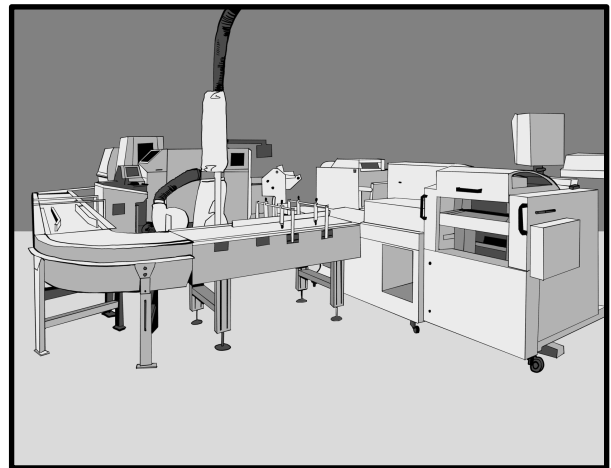
Use the following procedure to complete the decision matrix on the *Decision Matrix* student sheet.

Procedure:

1. For column #1-4 labeled with the company names, fill in the information designated in each row.
2. Color each square where you recorded data using the rules on the student sheet. The best scores are colored in green, the middle ones in yellow, and the worst in red.

When you make most purchases, there are difficult trade-offs that you need to consider. Does the machine package candy fast enough? Is accuracy or consistency going to be a problem? Is it very expensive?

3. In your group, analyze your decision matrices and decide which company you will buy your machine from.



There is not necessarily a right or perfect answer. However, you need to support your decision with evidence.



Discuss this question as a class:

1. Which company did your group pick, and why?

4.3 CONVINCING YOUR INVESTORS

Small business owners like you need to convince people with money—either banks or investors—to lend them money to buy major equipment. Sometimes people “pitch” ideas to investors to convince them. To do an effective pitch, you need data to support your argument.



Watch ***Shark Tank*** video #3.

On your *Pitch Planning* student sheet, plan your pitch for money from investors. Be sure to include the following information:

- What equipment you want
- How much it will cost
- Why you chose that particular company to purchase from
- Why should an investor loan money to you and your candy factory

**Georgia
Tech**  **Center for Education
Integrating Science,
Mathematics & Computing**

This curriculum is produced by Advanced Manufacturing & Prototyping Integrated to Unlock Potential (AMP-IT-UP) supported by National Science Foundation Award #1238089 through Georgia Institute of Technology's Center for Education Integrating Science, Mathematics, and Computing (CEISMC).

For more information about AMP-IT-UP
and to download our curriculum,
please visit our website at www.ampitup.gatech.edu.

